Intermittency of velocity fluctuations in turbulent thermal convection

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We analyze velocity fluctuations in turbulent Rayleigh-Bénard convection. The velocity measurements were taken at the center of an aspect-ratio-one convection cell filled with water. The measured probability density functions of the velocity difference over a time interval τ are found to change with τ, indicating that the velocity fluctuations are intermittent. The velocity intermittency can be well characterized by the She-Leveque hierarchical structure. Our analyses further show that the vertical velocity component has distinct statistical features from the horizontal components. This result indicates that the vertical direction is special and buoyancy is important even at the center of the convection cell.

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I. INTRODUCTION

Rayleigh-Bénard convection in an enclosed cell of fluid has been a model system for studying turbulence. The dynamics of the fluid is driven by an applied temperature difference across the height of the cell. The flow state is characterized by the geometry of the cell and two dimensionless numbers: the Rayleigh number $\mathrm{Ra}=\alpha g \Delta L^3/(\nu\kappa)$ and the Prandtl number $\mathrm{Pr}=\nu/\kappa$, where Δ is the applied temperature difference, L is the height of the cell, g is the acceleration due to gravity, and α, ν, and κ are, respectively, the volume expansion coefficient, kinematic viscosity, and thermal diffusivity of the fluid. When Ra is sufficiently large, convection becomes turbulent, and both the velocity and the temperature fields exhibit complex fluctuations both in time and in space. As in the study of other turbulent flows, a major issue is to make sense of these fluctuations. Another issue of interest specific to turbulent thermal convection is to understand the interplay between the velocity and temperature fields due to buoyancy. This is also pertinent to the questions of whether and how the statistical features of turbulence are affected by the presence of buoyancy. Using α, g and the average energy and temperature dissipation rates, ϵ and χ, one can construct a length scale known as the Bolgiano scale [1] $l_B \equiv \epsilon^{5/4}/[\chi^{3/4}(\alpha g)^{3/2}]$. When $g \to 0$, $l_B \to \infty$. Buoyancy effects are thus expected to be important in the Bolgiano regime of length scales larger than l_B.

For flows in which buoyancy is not driving the dynamics, Kolmogorov's 1941 theory [2] predicted that the velocity structure functions $\langle|\delta v_r|^p\rangle \equiv \langle|v(\vec{x}+\vec{r})-v(\vec{x})|^p\rangle$ scale as $r^{p/3}$ when $r\equiv|\vec{r}|$ is within the inertial range. Experimental measurements confirm the power-law behavior but indicate a nonlinear dependence of the scaling exponents on p. This deviation indicates that the velocity field has scale-dependent statistics, that is, the shape of the probability density function (PDF) of δv_r depends on r, and is intermittent. Extensive efforts have been devoted to understanding the intermittent nature of turbulence. Kolmogorov's refined similarity hypothesis (RSH) [3] attributed the intermittency of velocity fluctuations to spatial variations of the energy dissipation rate. She and Leveque [4] proposed a hierarchical structure for the moments of the local energy dissipation rate, which translates to a similar hierarchical structure for the velocity structure functions using RSH. This hypothesis for both the local energy dissipation rate and the velocity structure functions was supported by the velocity measurements taken in turbulent jets and wake [5–7]. It was also reported that the velocity structure functions in a class of shell models [8–10], the structure functions of a passive scalar [11], and the local passive scalar dissipation [12] all possess similar hierarchical structures.

In the study of turbulent Rayleigh-Bénard convection, many measurements of the local temperature fluctuations have been taken and analyzed. The temperature fluctuations were found to be intermittent [13]. Recently, a change in the scaling behavior of the normalized temperature structure functions was found [14] when the Bolgiano scale is crossed, indicating that buoyancy does affect the statistical characteristics of the temperature fluctuations. It was further found [15] that the intermittency of the temperature fluctuations in the Bolgiano regime can be attributed solely to the variations of the locally averaged temperature dissipation rate. That is, the temperature fluctuations conditioned at a given fixed value of the local temperature dissipation rate become Gaussian and thus scale independent. Moreover, both the temperature structure functions [14] and the local temperature dissipation rate [16] were found to satisfy the She-Leveque hierarchy.

On the other hand, velocity fluctuations in turbulent convection are much less studied. Recently, two of us (Qiu and Tong) have carried out direct velocity measurements in turbulent convection using the technique of laser Doppler velocimetry (LDV) [17,18]. The experiment was conducted in a cylindrical cell filled with water. The inner diameter of the cell is $D=19$ cm and its height is $L=20.4$ cm. In this paper, we report our analyses of the velocity time series data measured at the center of the convection cell. We concentrate on the measurements at the cell center because it is commonly believed that the turbulent convection in the central region is approximately homogeneous and isotropic. Results at a specific Ra, 3.7×10^9, will be discussed for the purpose.

In particular, we address two questions. First, are velocity statistics in turbulent convection different from those in flows in which buoyancy is not driven by dynamics? Second, is buoyancy important in the central region where there is no mean temperature gradient? We first investigate whether the velocity fluctuations are intermittent. We shall see in Sec. II that the answer is affirmative. Then, we proceed to characterize the intermittency of the velocity fluctuations in Sec. III. Specifically, we check whether the She-Leveque hierarchy is also a good description for the velocity fluctuations in turbulent convection. We shall also see that the answer is yes but the value of the parameter β is different from that reported for flows not driven by buoyancy. In Sec. IV, we show that the vertical component has distinct features from the other two horizontal components and discuss our interpretation that this result indicates that buoyancy is important in the central region. Finally, the work is summarized in Sec. V.

II. PROBABILITY DENSITY FUNCTIONS OF THE VELOCITY DIFFERENCE

We study the PDFs of the velocity difference over a time interval τ, $v_\tau \equiv v(t+\tau)-v(t)$, for the three velocity components, denoted as *x*, *y*, and *z* components. The *z* direction is the vertical direction, the *x* direction is defined by the direction of the mean large-scale flow observed near the bottom plate, and the *y* direction is fixed by the requirement that the *x*, *y*, and *z* axes form a right-handed coordinate system. Our aim is to investigate whether the shape of the PDF of v_τ changes with τ. For this purpose, we evaluate the PDFs of the standardized velocity difference, $X_\tau \equiv v_\tau/\langle v_\tau^2\rangle^{1/2}$ for different values of τ.

It is known that LDV has sampling bias towards higher velocity measurements [19]. To correct this bias, we use the minimum transit time of the two measurements, $v(t)$ and $v(t+\tau)$, as a weight when calculating the statistics of v_τ. As a result, the probability of v_τ having a value between x and $x+dx$ is estimated by the sum of the minimum transit times of those v_τ's with values in the same interval $(x,x+dx)$, divided by the total sum of the minimum transit times of all v_τ's.

Before presenting the results, let us first discuss some important time scales of the problem. Well-defined oscillations have been observed in the two horizontal velocity components [17] with an oscillation period τ_0 scaled as $L^2/(\kappa\tau_0)=0.2\ \mathrm{Ra}^{0.46}$ [20]. For $\mathrm{Ra}=3.7\times 10^9$, we have $\tau_0=55$ s. We associate this oscillation time τ_0 with the cell height L and define the time scale τ_B corresponding to the Bolgiano scale l_B by $\tau_B \equiv \tau_0 l_B/L$ [14]. In the definition of l_B, the average energy and temperature dissipation rates ϵ and χ were used. As a result, l_B can be written [21] in terms of Ra, Pr, L, and the Nusselt number (Nu) which is the heat flux normalized by that when there was only conduction. Thus, we have

$$\tau_B = \tau_0 \frac{\mathrm{Nu}^{1/2}}{(\mathrm{Ra\,Pr})^{1/4}}, \tag{1}$$

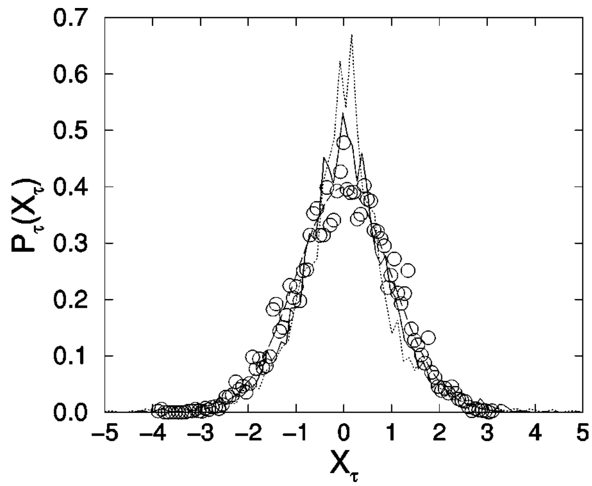

FIG. 1. Measured probability density functions $P_\tau(X_\tau)$ of the standardized velocity difference X_τ for the x component velocity. $\tau=1$ s (dotted line), $\tau=3.2$ s (solid line), and $\tau=26$ s (circles). The dashed line is a standard Gaussian distribution.

which is easily evaluated using the measured values of Ra, Pr, Nu, and τ_0. For $\mathrm{Ra}=3.7\times 10^9$, $\mathrm{Nu}=101$, and $\mathrm{Pr}=5.4$, we have $\tau_B \approx 1.5$ s. One can define a local version of the Bolgiano scale using instead the locally averaged energy and temperature dissipation rates. For example, using the energy and temperature dissipation rates averaged over a plane at a given height of the cell, a z-dependent Bolgiano scale $L_B(z)$ can be defined [22]. One expects that $l_B \approx L_B(z)$ when z is in the central region of the convection cell. The inverse of the cutoff frequency in the measured velocity power spectra [17] is $\tau_* \approx 1$ s. This sets the smallest time scale of interest for the velocity measurements. Note that τ_B is close to τ_*, and hence, only the Bolgiano regime can be probed with this set of velocity measurements.

In Fig. 1, we show the PDFs, $P_\tau(X_\tau)$, for the x-component velocity. It is seen that P_τ deviates from a Gaussian for small τ ($\tau \approx \tau_*$) and approaches a Gaussian as τ increases. Thus, the functional form of the PDF changes as τ increases. Hence, the fluctuations of the x-component velocity have scale-dependent statistics and are intermittent.

Similarly, we show $P_\tau(X_\tau)$ for the y- and z-component velocities in Figs. 2 and 3, respectively. Again the PDF changes its functional form as τ changes and approaches a Gaussian when τ approaches τ_0. Therefore, the y and z velocity components are also intermittent. Comparing Figs. 1–3, one finds that the change is more apparent for the vertical component than for the two horizontal components. We shall return to this in Sec. III.

III. CHARACTERIZATION OF THE VELOCITY INTERMITTENCY

We are interested in characterizing the velocity intermittency found in Sec. II. Specifically, we would like to check whether the She-Leveque hierarchical structure [4] is a good description for the velocity intermittency in turbulent thermal convection. For the velocity structure functions (in the time domain), $S_p(\tau)\equiv\langle |v_\tau|^p\rangle$, the hierarchical structure reads

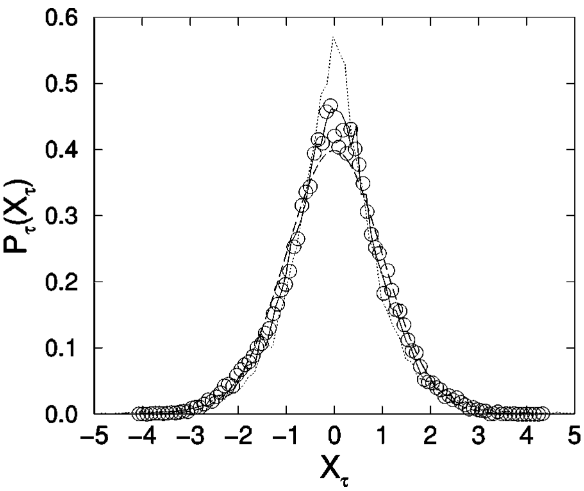

FIG. 2. $P_\tau(X_\tau)$ for the y-component velocity. Here, $\tau=1$ s (dotted line), $\tau=3.2$ s (solid line), and $\tau=50$ s (circles). The dashed line is a standard Gaussian distribution.

$$\frac{S_{p+2}(\tau)}{S_{p+1}(\tau)}=A_p\left[\frac{S_{p+1}(\tau)}{S_p(\tau)}\right]^{\beta}[S^{(\infty)}(\tau)]^{1-\beta}, \tag{2}$$

where $0<\beta<1$, A_p are constants independent of τ and

$$S^{(\infty)}(\tau)\equiv\lim_{p\to\infty}\frac{S_{p+1}(\tau)}{S_p(\tau)}. \tag{3}$$

If the PDF of v_τ is finite and is zero beyond a certain value, one can show that $S^{(\infty)}(\tau)$ is equal to the maximum value of $|v_\tau|$. Thus, for flows in which the velocity components are bounded, $S^{(\infty)}(\tau)$ exists, and is associated with the most intense structures of the flow. Suppose $S^{(\infty)}(\tau)$ scales with τ as $S^{(\infty)}(\tau)\sim\tau^b$ and $S_p(\tau)\sim\tau^{\zeta_p}$, then Eq. (2) implies

$$\zeta_p=a(1-\beta^p)+bp, \tag{4}$$

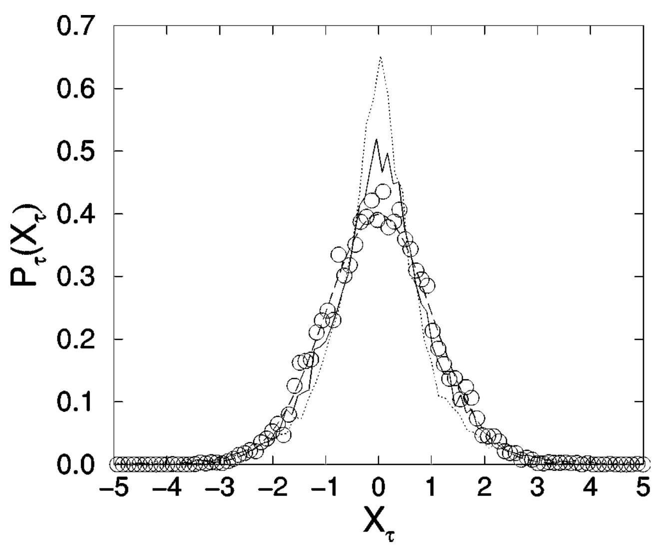

FIG. 3. $P_\tau(X_\tau)$ for the z-component velocity. Here, $\tau=1$ s (dotted line), $\tau=5$ s (solid line), and $\tau=50$ s (circles). The dashed line is a standard Gaussian distribution.

for some constant a. If $\beta\to1$, Eq. (4) gives $\zeta_p\to bp$ and we have ζ_p proportional to p, as predicted by Kolmogorov's 1941 theory. Thus, β can be taken as a measure of the degree of intermittency: the more β deviates from 1, that is, the smaller β is (since β is between 0 and 1), the more intermittent the fluctuation is.

It has been found [23] that the She-Leveque hierarchical structure is a special form of the general extended self-similarity (GESS) [24]. Thus, we first check whether GESS holds, that is, whether there exists relative scaling between the normalized structure functions:

$$T_p(\tau)\equiv\frac{S_p(\tau)}{[S_1(\tau)]^p}. \tag{5}$$

In Fig. 4, we plot $T_p(\tau)$ vs $T_2(\tau)$ for $\tau_*\leq\tau\leq\tau_0$ in a log-log scale for the three velocity components. The data points can be well fitted by a straight line indicating a good relative scaling. Thus, all the three velocity components have GESS, which holds in the whole range of τ for the vertical component but is restricted to a shorter extent (see Fig. 4) for the two horizontal components due to the oscillations.

We then follow the procedure developed in Ref. [23] to check whether this GESS is of the She-Leveque form. The first step is to find the relative scaling exponents $\rho(p)$, defined by

$$T_p(\tau)\sim[T_2(\tau)]^{\rho(p)}, \tag{6}$$

and study $\Delta\rho(p)\equiv\rho(p+1)-\rho(p)$. If Eq. (2) holds, then one can show that

$$\Delta\rho(p+1)=\beta\Delta\rho(p)+1. \tag{7}$$

The second step is to plot $\Delta\rho(p+1)$ vs $\Delta\rho(p)$ and check whether the data points can be fitted by a straight line of intercept 1. In Fig. 5, we show such plots for the three velocity components. It is seen that straight lines with intercept 1 can indeed fit all three sets of data. Hence, we conclude that the She-Leveque hierarchical structure is a good description for all three velocity components. The value of β is given by the slope of the fitted straight line. We find an interesting result that the data for the two horizontal components can be well fitted by the same straight line with the same value of $\beta=0.79\pm0.03$. On the other hand, we find $\beta=0.66\pm0.02$ for the vertical velocity component. These values of β are smaller than the value of 0.87 [4,7] for velocity fluctuations in turbulent flows in which buoyancy is not driving the dynamics. Thus, velocity fluctuations in turbulent convection are generally more intermittent.

IV. COMPARISON OF THE STATISTICAL FEATURES OF THE THREE VELOCITY COMPONENTS

In the preceding section, We have shown that the vertical velocity component has a smaller value of β than the two horizontal components have and thus has more intermittent fluctuations. This is consistent with our observation in Sec. II that the change of the PDF $P_\tau(X_\tau)$ with τ is more apparent for the vertical velocity component. Figure 5 thus suggests

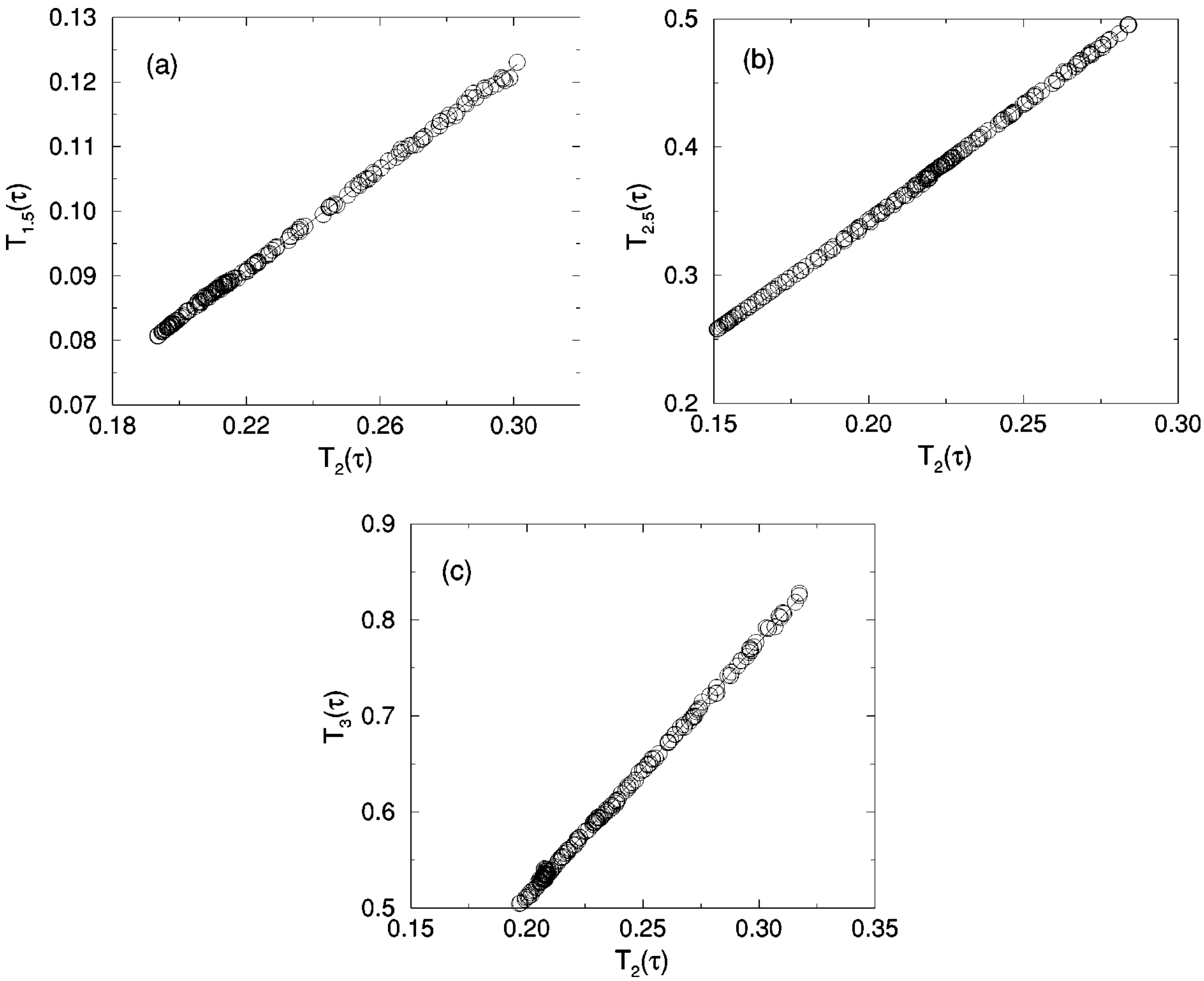

FIG. 4. $T_p(\tau)$ vs $T_2(\tau)$ for (a) the *x*-component velocity with $p=1.5$, (b) the *y*-component velocity with $p=2.5$, and (c) the *z*-component velocity with $p=3$. The solid line is the best linear fit for the region that can be fitted by a straight line and extrapolated to the whole region of τ to show the extent of the GESS range.

that the vertical component is distinct from the two horizontal velocity components.

We shall further explore this issue by studying the behavior of $S^{(\infty)}(\tau)$ for the three velocity components. We follow

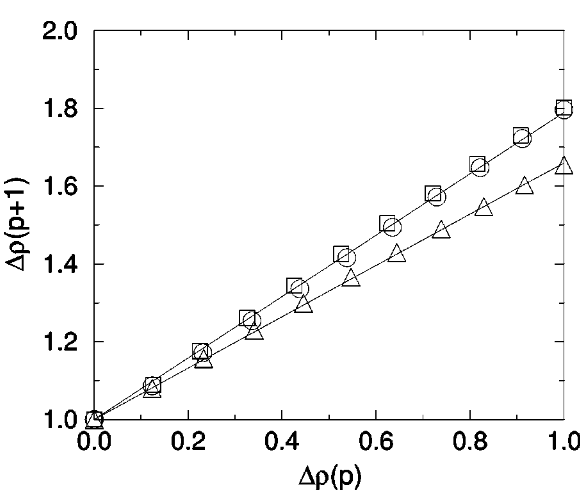

FIG. 5. $\Delta\rho(p+1)$ vs $\Delta\rho(p)$ for the *x*-component (circles), *y*-component (squares), and *z*-component (triangles) velocities.

the method described in Ref. [23] to get an indirect estimate of $S^{(\infty)}(\tau)$. First, we define $F_p(\tau)$ by

$$F_p(\tau)=\frac{\log_{10}[S_p(\tau)/S_p(\tau_*)]-f(p)\log_{10}[S_1(\tau)/S_1(\tau_*)]}{p-f(p)}, \tag{8}$$

where $f(p)=(1-\beta^p)/(1-\beta)$. From Eq. (2), $F_p(\tau)$ should be independent of p and equal to $\log_{10}[S^{(\infty)}(\tau)/S^{(\infty)}(\tau_*)]$. Figure 6 shows $F_p(\tau)$ for the three components. The data for each component collapse when p is large as expected. Oscillations reported in the two horizontal components again show up in $F_p(\tau)$. For the z component, $F_p(\tau)$ becomes approximately independent of τ when $\tau\geqslant 5$ s. On the other hand, for the horizontal components $F_p(\tau)$ exhibits a clear τ dependence. Thus, $S^{(\infty)}(\tau)$ depends on τ for the horizontal components but is consistent with being τ independent for the vertical velocity component when $\tau\geqslant 5$ s. This result suggests that the most intense structures associated with the vertical velocity are shocklike [23]. A further consequence of this result is the saturation of ζ_p for the vertical velocity

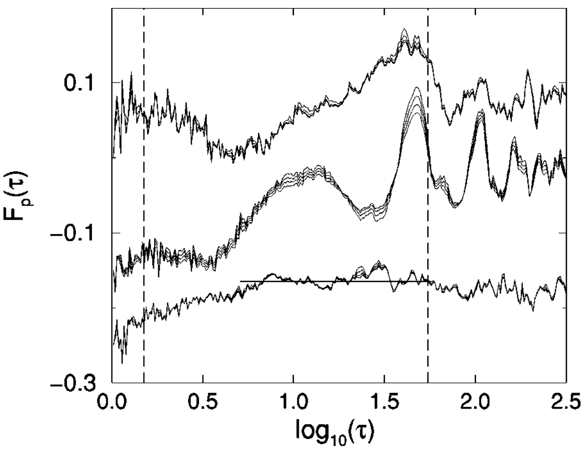

FIG. 6. $F_p(\tau)$ with $p=1.5$, 2, 2.5, and 3 for the *x*-component, *y*-component, and *z*-component velocities (from top to bottom). The data for the *y* and *z* components are shifted downwards by 0.15 and 0.25, respectively, for clarity.

structure functions as $p\to\infty$ [see Eq. (4)]. Hence, the vertical component again shows distinct statistical features from the two horizontal velocity components.

Our analyses reveal that the vertical direction is special even at the cell center. On the one hand, this result is in accord with the presence of buoyancy, which drives the fluid dynamics in turbulent thermal convection. On the other hand, because the mean temperature gradient vanishes in the central region of the convection cell, it is commonly believed that buoyancy does not play a role there, and this result is, therefore, not totally expected. When buoyancy becomes important, one expects that hot fluid rises and cold fluid falls and hence a positive correlation between the vertical velocity and temperature fluctuations is produced. The correlation between v_τ and $T_\tau \equiv T(t+\tau)-T(t)$ has been recently studied and reported in Ref. [25]. Indeed a positive correlation is found for the vertical velocity component when $\tau \geqslant \tau_B$, and only a weak correlation for the two horizontal components is observed throughout the whole range of τ [25]. This result thus suggests that buoyancy is important even at the center of the cell, which reinforces the present finding that the vertical direction is special even at the cell center and that the vertical velocity component is distinct from the horizontal velocity components. The positive correlation between the vertical velocity and temperature differences also suggests that there are plumes passing through the central region. These plumes will likely cause the vertical velocity fluctuations to be more intermittent than the horizontal velocity fluctuations as found in Sec. III.

V. SUMMARY

We have carried out a systematic study of the statistical properties of velocity fluctuations in turbulent thermal convection. We focused on the velocity measurements taken at the center of the convection cell. It is first established that fluctuations of all the three velocity components have scale-dependent statistics and are thus intermittent. It is found that the velocity intermittency in turbulent thermal convection can be well described by the She-Leveque hierarchical structure. Our analyses show that the velocity fluctuations in turbulent thermal convection are generally more intermittent than those in turbulent flows that are not driven by buoyancy. Finally, our analyses reveal that the vertical direction is special even at the center of the cell. This is reflected by the finding that the vertical component has distinct features from the two horizontal components. First, the vertical component has a smaller value of β and thus has more intermittent fluctuations. Second, the most intense structures associated with the vertical component are shocklike and the scaling exponents ζ_p of the vertical velocity structure function saturate as $p\to\infty$. Together with the finding of a positive correlation between the vertical velocity and temperature differences [25], this result indicates that buoyancy is important even in the central region of the cell where there is no mean temperature gradient.

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